Dynamic Type Matching

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Emerging Applications

Car Hailing

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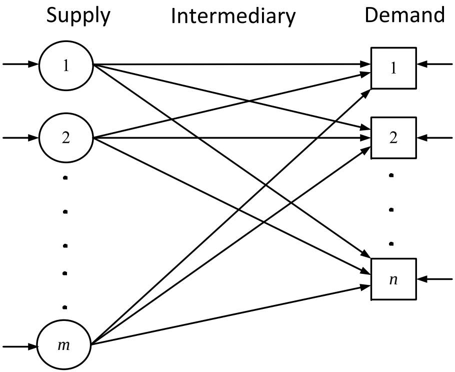
When is the greedy matching optimal?

Model Features



- Centralized matching by a platform

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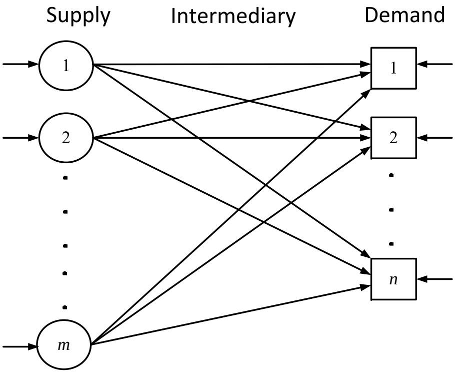

- Centralized matching by a platform
- Inter-temporal uncertainty

Emerging Applications: e-Commerce

- Amazon: inventory commingling program

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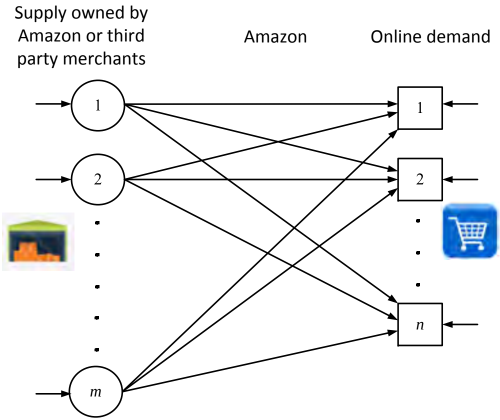

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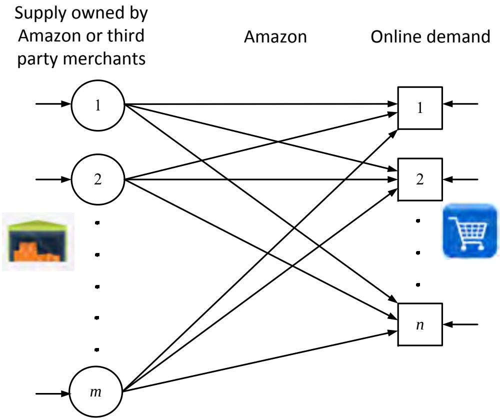
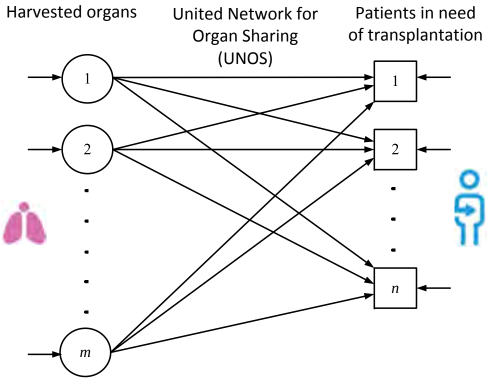


Types: geographic locations (horizontally differentiated)
"idiosyncratic" preference

Emerging Applications: Organ Transplant

- Kidney allocation Zenios et al. 2000, Su and Zenios 2005
- Liver allocation Akan et al. 2014



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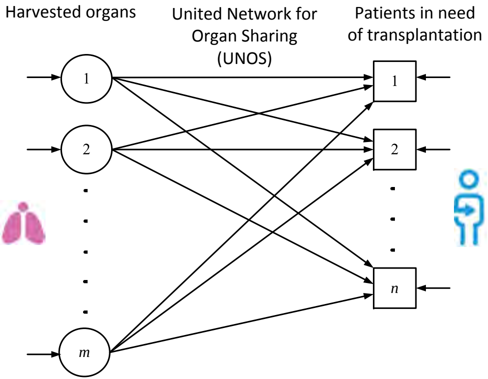

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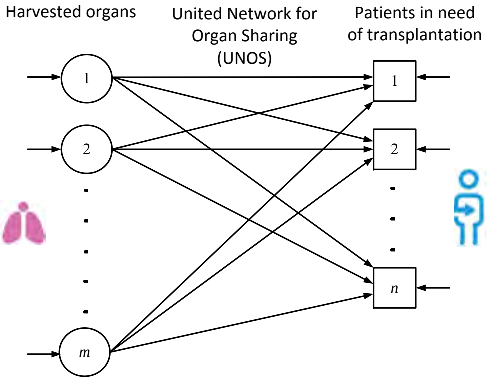

Types: health status (vertically differentiated)

"uniform" preference

blood/tissue (horizontally differentiated)

The Model

- An intermediary firm matches:
    - Demand types $\mathcal{D} = \{1, 2, \ldots, n\}$, indexed by i
    - Supplier types $\mathcal{S} = \{1, 2, \ldots, m\}$, indexed by j

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- Random arrivals in a period with arbitrary distributions
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  - Demand $\mathbf{D} = (D_1, \ldots, D_n)$
  - Supply $\mathbf{S} = (S_1, \ldots, S_m)$
- Decisions, revenue and costs
  - Decisions: matching quantity q_{ij} ($\mathbf{Q}$)
  - Unit reward r_{ij} ($\mathbf{R}$)
  - Unit holding cost c and h for unmatched demand and supply, resp.
- Unmatched demand and supply carry over to the next period with rates α and β, resp.

Stochastic Dynamic Program

- State variables $(\mathbf{x}, \mathbf{y})$, after arrival before matching
  - $\mathbf{x}$: demand levels
  - $\mathbf{y}$: supply levels
- Post matching levels $(\mathbf{u}, \mathbf{v})$, after matching
  - $\mathbf{u} = \mathbf{x} - \mathbf{1}^m \mathbf{Q}^\top$ and $\mathbf{v} = \mathbf{y} - \mathbf{1}^n \mathbf{Q}$
- Optimal recursion

$$
\begin{aligned}
V_t(\mathbf{x}, \mathbf{y}) &= \max_{\mathbf{Q} \in \{\mathbf{Q} \geq \mathbf{0} | \mathbf{u} \geq \mathbf{0}, \mathbf{v} \geq \mathbf{0}\}} H_t(\mathbf{Q}, \mathbf{x}, \mathbf{y}), \\
H_t(\mathbf{Q}, \mathbf{x}, \mathbf{y}) &= \mathbf{R} \circ \mathbf{Q} - c \mathbf{1}^n \mathbf{u}^\top - h \mathbf{1}^m \mathbf{v}^\top \\
&\quad + \gamma E V_{t+1}(\alpha \mathbf{u} + \mathbf{D}, \beta \mathbf{v} + \mathbf{S}) \\
V_{T+1}(\mathbf{x}, \mathbf{y}) &= 0
\end{aligned}
$$

Classic Settings

- Capacity management with upgrading Shumsky and Zhang (2009), Yu et al. (2015)
- Centralized matching market e.g., medical residence
- Inventory rationing
- Assignment/transportation problem
- Type mating Duenyas et al. (1997)

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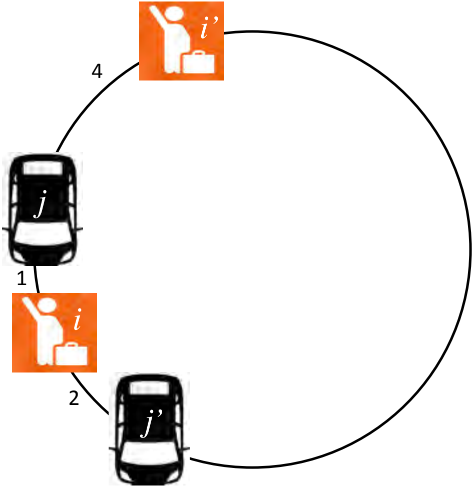
  - General priority properties under modified Monge condition
    - Sufficient, and robustly *necessary*
  - Vertically differentiated types
    - Quality-based priority
  - Horizontally differentiated types
    - Distance-based priority
  - Bounds and heuristics

Wh



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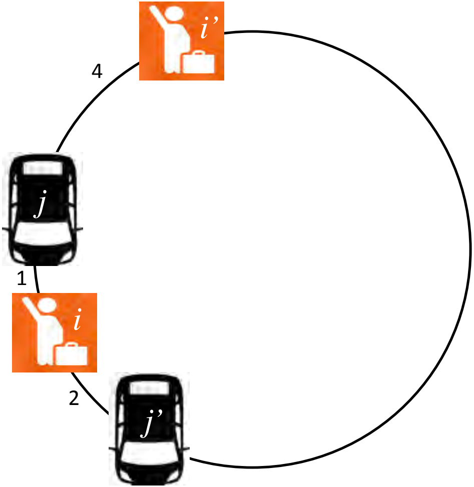


Greedy matching is not optimal!

A Relation of Neighboring Arcs

Definition (Modified Monge Condition)

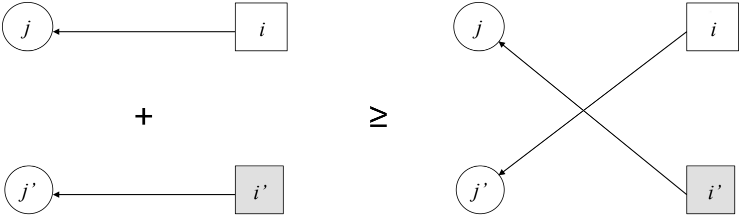
We say $(i, j) \succeq (i, j')$, *if*

(i) $r_{ij} \geq r_{ij'}$

(ii)

$$r_{ij} + r_{i'j'} \geq r_{ij'} + r_{i'j} \quad \text{(D)}$$

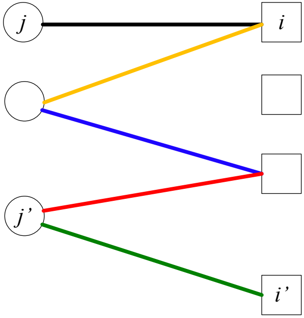
for all $i' \in \mathcal{D}$.



A Partial Order between Arcs

Definition (Arcs without common nodes)

For $i \neq i'$ and $j \neq j'$, we say $(i,j) \succeq (i',j')$ if there exists a decreasing sequence of neighboring arcs connecting the two.



Greedy Matching for a Perfect Pair

Theorem (When Greedy Matching is Optimal)

If $(i,j) \succeq (i,j')$ *for all* $j' \in \mathcal{S}$ *and* $(i,j) \succeq (i',j)$ *for all* $i' \in \mathcal{D}$,

$$q^*_{ij} = \min\left\{x_i, y_j\right\}.$$

Priority Hierarchy

Theorem

There exists an optimal decision $\mathbf{Q}^*$ *such that for any* $(i,j) \succ (i',j')$,

$$(i,j) \text{ has a higher priority to be matched over } (i',j').$$

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- The proof generalizes the augmenting path approach to DP
- We do not require all neighboring arcs are comparable
- For horizontal and vertical cases, all neighboring arcs are indeed comparable

Monge Sequence

By Gaspard Monge in 1781

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Comparison with Monge Sequence

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Comparison with Monge Sequence

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a sequence	pairs
static, *deterministic* and *balanced* transportation problem	*dynamic*, *stochastic* and *unbalanced* transportation problem
sufficient and necessary	sufficient, and *robustly* necessary
a greedy algorithm: (1) priority property (2) match as much as possible	Our result: (1) priority property (2) match-down-to policy

Vertically Differentiated Types

- Decomposable reward:

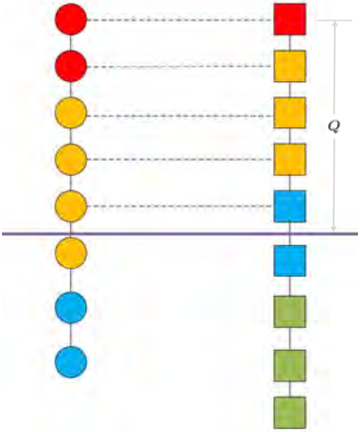
$$r_{ij} = r_i^d + r_j^s$$

- Centralized medical residency assignment Agarwal (2015)

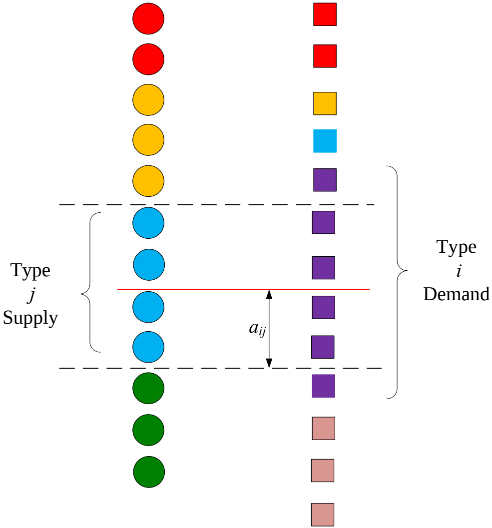
Vertical Model: Optimal Policy

Top-down matching:

- Line up demand and supply from high to low
- Match up from the top (to some level)



Vertical Model: Optimal Policy (Dynamic View)



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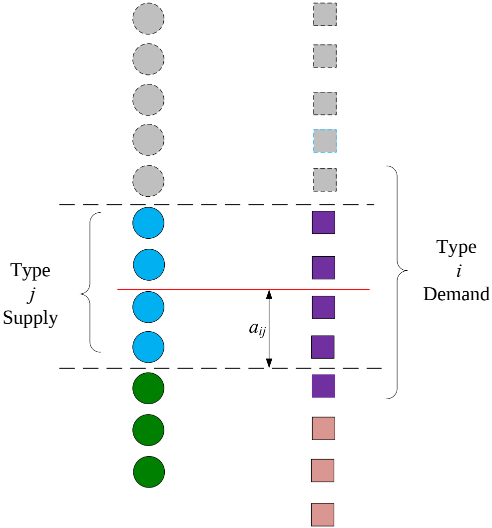

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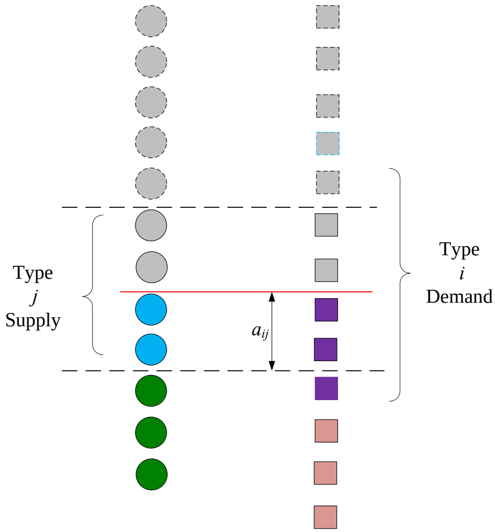
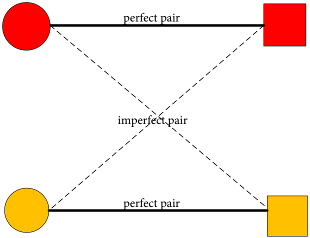


Horizontal Model: 2-to-2 Case

- $n = m = 2$
- $r_{ii} \geq \max\{r_{i,-i}, r_{-i,i}\}$ for $\{i, -i\} = \{1, 2\}$



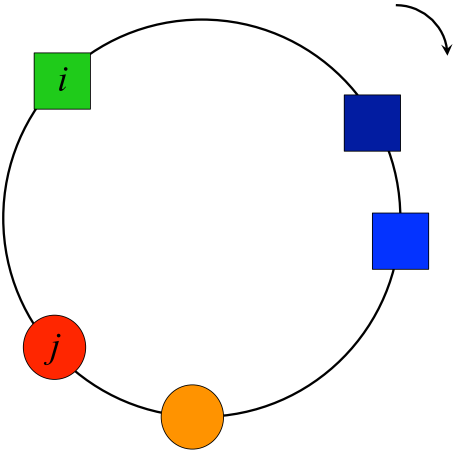
Horizontally Differentiated Types

Horizontal Model: Optimal Policy of 2-to-2 Case

Proposition

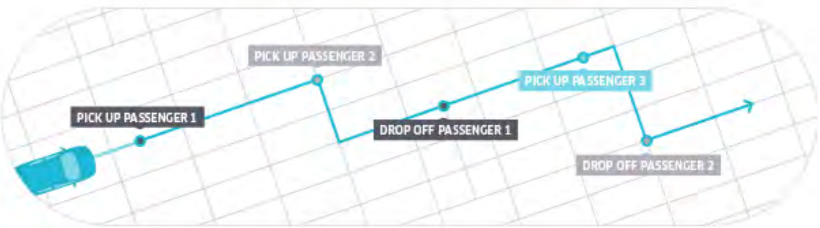
- *Step 1. Greedy matching for the perfect pair: Match type i demand with type i supply as much as possible,* $i = 1, 2$
- *Step 2. Match-down-to policy for the imperfect pair: Match type i demand with type* $-i$ *supply only when* $\eta_i \equiv x_i - y_i > 0$ *and* $\eta_{-i} \equiv x_{-i} - y_{-i} < 0$
  - *The remaining quantity of type i demand and type* $-i$ *supply after Step 1:* η_i *and* $-\eta_{-i}$*, resp.;* $q^*_{-i,i} = 0$
  - *The optimal protection level* $\bar{a}_{it}(\eta) \geq 0$ $(\eta \equiv \eta_i + \eta_{-i})$
    - *If* $\eta_i \geq \eta^+ + \bar{a}_{it}(\eta)$*, then reduce type i demand to* $\eta^+ + \bar{a}_{it}(\eta)$*, type* $-i$ *supply to* $\eta^- + \bar{a}_{it}(\eta)$
    - *If* $\eta_i < \eta^+ + \bar{a}_{it}(\eta)$*, do not match type and set* $q^*_{i,-i} = 0$

Horizontally Differentiated Types



$r_{ij} = f(d_{ij})$, where d_{ij} is the clockwise distance between i and j

Logistics with Fixed Routes in the Same Direction



UberPool

More Emerging Applications

Load Matching

Horizontal Model: Car Pooling

Horizontal Model: Priority by Distance

Theorem (Greedy Match of Perfect Pair)

Suppose that type i demand and type j supply are closest to each other. If f is nonincreasing and convex, $q_{ij}^* = \min\{x_i, y_j\}$.

Theorem (Distance-Based Priority of Imperfect Pairs)

If f is nonincreasing and linear, for any given type i demand,

- *the closer its distance to a type j supply, the higher the priority in matching the demand-supply pair* (i, j);
- *Along the priority hierarchy, the optimal matching is a match-down-to policy.*

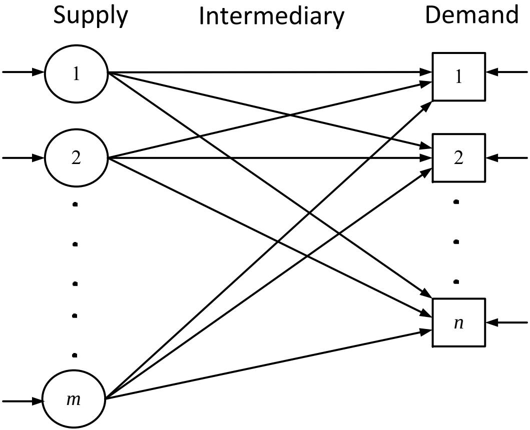
Deterministic Heuristic for the General Problem

- The deterministic model provides an upper bound for the stochastic model
- Successively resolving the deterministic model is asymptotically optimal for the stochastic model

Extensions

- Time-dependent parameters
- Type-dependent parameters, e.g., $c_1 \geq \cdots \geq c_n$, $h_1 \geq \cdots \geq h_m$
- Random abandonments
- Forbidden arcs
- Forced maxing-out
- A continuum of types
- Infinite horizon with discounted or long-run average payoff
- Other forms of r_{ij}
  - $r_{ij} = \min \left\{ r_i^d, r_j^s \right\}$
  - $r_{ij} = \max \left\{ r_i^d, r_j^s \right\}$
- Endogenized supply process and pricing

Summary



A New Form of Matching Supply with Demand

- Operations Management manages the process of matching supply with demand
- Foundations
  - Inventory management (e.g., base-stock policy)
  - Revenue management (e.g., protection level)
- New form of business process

Matching in a two-sided market with crowdsourced supply
(sharing economy)

Summary: Distribution-Free Structural Results

- General priority properties under modified Monge condition
  - Sufficient, and robustly *necessary*
- Vertically differentiated types
  - Quality-based priority+match-down-to policy
- Horizontally differentiated types
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Thank you!