Peer-to-Peer Trading of Usage Quotas

Behrooz Pourghannad
University of Minnesota

Joint work with Saif Benjaafar and Jian-Ya Ding

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Quota Based Business Models

Software as a service

fractional ownership of private jets

access to various gyms and fitness classes

KarmaGo, a WiFi provider, allows users to share their quota

Event tickets:
Access to a particular seat during the event

fractional ownership of vacation houses
Mobile Hotspot Sharing

- Developed by MIT Media Lab in 2013
- “Air Mobs creates a local mobile community to allow users to freely share internet access.”

Challenge

“There is a good chance that Verizon, AT&T, and other stakeholders would come in and shut it down.”
A trading platform for unused capacity.
- 2cm, started 2013

Two plans
- 1GB of data
- 5GB of data

“Many users who subscribe to a 5-GB monthly plan may not use all the data. 2cm data exchange platform would give them the chance to trade capacity.”

– China Mobile Hong Kong chairman Tiger Lin Zhenhui
Firm Initiated Capacity Sharing

Some firms not only allow peer-to-peer trading, but also provide infrastructure to facilitate peer-to-peer capacity trading.

Share your Ford car using **Ford car sharing** platform

**Toyota keyless system**: unlock doors and start cars with a smartphones
Research Questions

Is allowing customers to share their excess capacity beneficial?

- How is the service provider’s profit affected?
- How is the customers’ surplus affected?
  - Individual
  - Aggregate
- What will happen to aggregate consumption?
- How is social welfare affected?
- What is the optimal contract design in the presence of peer-to-peer trading?
Related Literature

Secondary market for used durable goods
  — value enhancement vs. market cannibalization
  — Common wisdom: the secondary markets improves social welfare by increasing allocation efficiency and negatively affects provider’s profit


Peer-to-peer product sharing
Model Setup

- Two quota plans
- Risk-neutral monopolist
- Service cost $c$ per unit of usage

Assumptions

A1: Full market coverage
A2: Full plan coverage

- Continuum of customers
- Types, denoted by $\theta$, are uniformly distributed on $[0, 1]$.
- Type-dependent consumption such that consumption is either $H$ or $L$ with

\[
\mathbb{P}(x = H) = \theta \\
\mathbb{P}(x = L) = 1 - \theta
\]
Without Trading vs. with Trading

The service provider decides on the contract terms

Customers select a contract

Each customer learns his own demand

Customers use the platform to buy/sell data at price $\pi$

Customers’ surplus

\[ u_n(q, \theta) = r \mathbb{E}(x|x \leq q) - p \]

\[ u_t(q, \theta) = u_n(q, \theta) + \alpha \pi \mathbb{E}(q - x|x \leq q) + \beta (r - \pi) \mathbb{E}(x - q|x > q) \]

\[ \alpha = \min \left\{ \frac{\text{total demand}}{\text{total supply}}, 1 \right\} \]

\[ \beta = \min \left\{ \frac{\text{total supply}}{\text{total demand}}, 1 \right\} \]

Overage

Underage

$\beta = 1 \Rightarrow \alpha \leq 1$

$\alpha = 1 \Rightarrow \beta \leq 1$
There exists a unique equilibrium in the form of a switching strategy

$$\theta_n = \frac{\delta}{r}, \quad \theta_t = \begin{cases} \frac{\delta}{\delta + \pi} & \text{if } \delta \leq \pi \leq r \\ \frac{\pi}{2r - (\delta + \pi)} & \text{if } 0 \leq \pi \leq \delta \end{cases}$$

with $\delta = \frac{p_2 - p_1}{q_2 - q_1}$

Let $m_1^t = (L, p_1)$ and $m_2^t = (H, p_2)$, then the service provider solves

$$\max_{p_1^t, p_2^t} p_1^t \theta_t + p_2^t (1 - \theta_t) - c \, P_t$$

subject to

$$u_t(\hat{\theta}_1, m_1^t) \geq u_t(\hat{\theta}_1, m_2^t) \quad \text{for some } \hat{\theta}_1 \in [0,1]$$

$$u_t(\hat{\theta}_2, m_2^t) \geq u_t(\hat{\theta}_2 m_1^t) \quad \text{for some } \hat{\theta}_2 \in [0,1]$$

$$u_t(0, \hat{m}) \geq 0 \quad \text{for some } \hat{m} \in \{m_1^t, m_2^t\}$$
The Service Provider’s Profit

The Service Provider’s profit increases by allowing peer-to-peer trading of usage quota if and only if the trading price $\pi$ is higher than the threshold $\pi^t_1$. Moreover, the threshold $\pi^t_1$ is unique and increases in $c$. 

![Graph showing service provider's profit with and without trading against trading price for different values of c.]

- $c = 0.2$
- $c = 0.4$
- $c = 0.6$
- $c = 0.8$
1. There is $\theta_\ell \in [0,1]$ such that all customers of types $\theta \geq \theta_\ell$ are better off and all customers of type $\theta < \theta_\ell$ are worse off.

2. The customer of type $\theta_n$ benefits the most
Aggregate Customers’ Surplus

The aggregate customers’ surplus increases, by peer-to-peer trading, if and only if $\pi_3 \leq \pi \leq \pi_4$. Moreover, both thresholds increase in $c$. 

![Graph showing the aggregate customers' surplus vs. trading price for different values of c.]
Social Welfare

Peer-to-peer trading leads to higher social welfare if and only if the trading price is higher than the unique threshold $\pi_2^S$. Moreover, $\pi_2^S < \pi_1^S$ and $\pi_2^S$ increases in $c$.

— If the market is such that the trading price is kept sufficiently high, then sharing is socially optimal.
Summary of Results

— Peer-to-peer trading creates a win-win situation if the trading price is moderate

— Interestingly, if the trading price is low enough the aggregate customers’ surplus decreases by the peer-to-peer trading

— If the trading price is sufficiently high, both the service provider’s profit and the social welfare are improved by peer-to-peer trading

<table>
<thead>
<tr>
<th>Agg. Customers’ Surplus</th>
<th>Lower</th>
<th>Higher</th>
<th>Higher</th>
<th>Lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provider’s Profit</td>
<td>Lower</td>
<td>Lower</td>
<td>Higher</td>
<td>Higher</td>
</tr>
<tr>
<td>Social Welfare</td>
<td>Lower</td>
<td>Higher</td>
<td>Higher</td>
<td>Higher</td>
</tr>
</tbody>
</table>

Win-Win

\[ \pi_4^t \quad \pi_3^t \quad \pi_1^t \quad \pi_2^t \]

Trading Price
— All customers are better off and the aggregate customers’ surplus is always higher

— If the trading price is sufficiently high, both the service provider’s profit and the social welfare are improved by peer-to-peer trading

— The win-win condition is characterized by only a lower bound on the trading price
Main Contributions and Takeaways

• A model of trading unused capacity among customers

• Insights into the impact of trading on consumer surplus, provider’s profit, and social welfare

• Characterize the condition under which trading unused capacity is win-win with respect to performance metrics we introduced

• Our results provide a framework for firms and social planners to evaluate the profitability of peer-to-peer trading